

ASPECTS OF NEUTRINO COSMOLOGY *

KARI ENQVIST

Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

ABSTRACT

Primordial nucleosynthesis constrains the properties of light, stable neutrinos. Apart from the well-known limit on the number of neutrino species, there are also bounds on neutrino masses and magnetic moments. I discuss also sterile neutrinos and neutrino propagation in a primordial magnetic field, such as could be the origin of the observed galactic magnetic fields.

1. Introduction

The LEP results have made it clear that there are only three stable light left-handed neutrinos which couple to Z. Neutrinos might be exactly massless, but even if they have masses, we do not know whether they should be Dirac or Majorana masses. For ν_e there exist several mass measurements¹, all yielding $m_{\nu_e} \lesssim 10$ eV (although the average $\langle m_{\nu_e}^2 \rangle$ is negative). The mass limit on ν_μ has recently been revised upward¹ to 500 keV, although it is likely to be improved soon. The current limit on ν_τ mass is $m_{\nu_\tau} \leq 31$ MeV². If neutrino masses are of the Majorana type, the non-observation of neutrinoless double beta decay $(Z, A) \rightarrow (Z + 2, A) + 2e^-$ implies that³ $\langle m_\nu \rangle \equiv \sum U_{ei}^2 m_{\nu_i} \lesssim 1.2$ eV; barring accidental cancellations between the mixing matrix elements U_{ei} , the Majorana mass of ν_e should thus be less than about 1 eV.

If neutrinos are Dirac particles, in the Standard Model they will have small one-loop induced magnetic moments, given by $\mu_\nu = 3.1 \times 10^{-19} (m_\nu/\text{eV}) \mu_B$ where μ_B is the Bohr magneton. Studies of $\nu_e e^- \rightarrow \nu_e e^-$ and $\nu_\mu e^- \rightarrow \nu_\mu e^-$ elastic scattering can be used to derive the experimental bounds⁴ $\mu_{\nu_e} < 1.1 \times 10^{-9} \mu_B$

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and $\mu_{\nu_\mu} < 7.4 \times 10^{-10} \mu_B$. The best limit on μ_{ν_τ} is obtained from D decays in beam dump experiments⁵, yielding $\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B$. Majorana neutrinos cannot have diagonal magnetic moments, but if neutrinos mix, they will have non-zero transition magnetic moments.

There are a number of astrophysical constraints on neutrino properties, mainly based on cooling of stars and the supernova SN1987A. Energy would be transferred too fast from the inner core of the supernova if left-handed neutrinos flip over to right-handed neutrinos. These would then freely stream out⁶, thus affecting the energetics of the supernova. The most recent numerical study⁷ employs a supernova code that includes also neutrino-nucleon scattering and has a higher meson density in the core compared with previous estimates. Therefore the importance of processes like $\pi + N \rightarrow N + \nu\bar{\nu}$ is enhanced, and one obtains a rather stringent upper limit of 3 keV on the neutrino Dirac mass term.

There is also the well-known cosmological limit on the sum of all stable neutrino masses, whether Dirac or Majorana, which can be obtained by requiring that the relic neutrino mass density does not exceed the upper limit of the density of the universe. One finds

$$\sum m_{\nu_i} = 92\Omega_{0\nu}h^2 \text{ eV} , \quad (1)$$

where $\Omega_{0\nu}$ is the energy density of neutrinos today, in units of the critical density, and h is the Hubble parameter in units of $100 \text{ kms}^{-1}/\text{Mpc}$. The age estimates of the universe imply⁸ that $\Omega_0 h^2 \lesssim 0.25$ so that $\sum m_{\nu_i} \lesssim 23 \text{ eV}$. If there exists a cosmological constant, then the limit is somewhat relaxed; for $\rho_{\text{vac}} = 0.8\rho_{\text{crit}}$ one obtains $\sum m_{\nu_i} \lesssim 35 \text{ eV}$.

For cosmology the difference between Dirac and Majorana neutrinos is significant because that difference may be crucial for primordial nucleosynthesis of light elements⁹. If neutrinos are Dirac particles, then the right handed chirality state ν_R , as well as the left-handed antineutrino $\bar{\nu}_L$, must exist. This means that at very high temperatures, for each neutrino flavour, there were four spin-degrees of freedom in equilibrium (in the Standard Model equilibration of right-handed neutrinos takes place e.g. through the one-loop induced magnetic moment). Later right-handed neutrino interactions, being very weak, decoupled and their number densities were diluted by subsequent annihilations. It is then essential that this decoupling occurs at high enough temperature so that the relic density of right-handed neutrinos at the onset of primordial nucleosynthesis

is small enough. The current nucleosynthesis limit on the maximum number of extra degrees of freedom, quantified in units of two-component massless fermions, is often quoted as¹⁰

$$\delta N_\nu \simeq 0.3 \quad (2)$$

This obviously sets a limit on the strength of interactions which turn left-handed neutrinos into right-handed ones. These interactions will necessarily involve spin-flip operators such as the mass and the magnetic moment.

More generally, nucleosynthesis imposes a constraint on the equilibration rate of any sterile neutrino, be it left- or right-handed. This turns out to be a very useful way to limit oscillations between sterile and active neutrino species.

2. Primordial nucleosynthesis

At temperatures $T \gg \mathcal{O}(1)$ MeV neutrons and protons were kept in chemical equilibrium through the weak processes $\nu_e n \leftrightarrow pe$, $en \leftrightarrow p\bar{\nu}_e$ and $n \leftrightarrow pe\bar{\nu}_e$. At that time the relative number of neutrons was simply given by the equilibrium ratio

$$X(T) \equiv \frac{n_n(T)}{n_n(T) + n_p(T)} = \left(1 + e^{\Delta m/T}\right)^{-1}, \quad (3)$$

where $\Delta m = m_n - m_p$. The neutron-to-proton ratio froze out at about $T \simeq 0.7$ MeV, after which the neutron population was still reduced by free neutron decay until $T \simeq 0.1$ MeV, at which point photons no longer were energetic enough to prevent protons and neutrons to combine to form deuterium. Consequently reactions like $D + D \rightarrow {}^3\text{He} + n$, $T + p$; $D + (p, T, {}^3\text{He}) \rightarrow {}^3\text{He} + \gamma$, ${}^4\text{He} + p(n)$ and ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p$ helped to build up about 25% ${}^4\text{He}$ and traces of ${}^3\text{He}$, D and heavier elements such as lithium and beryllium. Therefore, assuming the neutron life-time is known, the primordial abundance of ${}^4\text{He}$ reflects essentially directly the neutron-to-proton ratio at the freeze-out, which depends only on the ratio $\Gamma_{n \leftrightarrow p}(T)/H(T)$, where $H(T)$ is the Hubble rate.

The theoretical Helium abundance thus depends mainly on two things: the number of degrees of freedom at the freeze-out which appear in the Hubble rate, given by $H = (8\pi^3 g_*(T)/90)^{1/2} T^2/M_{Pl}$ where $g_*(T) = \sum g_B(T) + \frac{7}{8}g_F(T)$ is the effective number of degrees of freedom; and the number density of electron neutrinos, which can affect the neutron-to-proton rate $\Gamma_{n \leftrightarrow p}(T)$. In addition,

one needs to know the baryon density of the universe, as well as the neutron life-time, which recently has been revised⁹ to 10.26 ± 0.03 min. There is also some inherent uncertainty in the nuclear reaction rates, but for ${}^4\text{He}$ the main uncertainty comes from neutron life-time. A fit to data then yields¹⁰

$$N_\nu = 3.0 - 0.8 \ln \eta_{10} + 19 \left(\frac{Y_p - 0.228}{0.228} \right) - 15 \left(\frac{\tau_n - 889.8}{889.8} \right), \quad (4)$$

where Y_p is the primordial ${}^4\text{He}$ -abundance, τ_n is the neutron life-time in seconds and $\eta_{10} \lesssim 2.8$ is the baryon number in units of 10^{-10} .

Regression analyses of He mass fraction against O and N abundances, with 1σ limits. Larger and smaller circles represent higher and lower weights, respectively, open circles are objects not enriched by Wolf-Rayet stars, and a few typical error bars are shown (from ref. 11).

The primordial abundances of light elements can be deduced from observations in various ways. For instance, one measures the relative ${}^4\text{He}$ -abundance in extragalactic ionized hydrogen regions with different metallicities. The result is then interpolated to zero metallicity, which implies¹¹ a primordial abundance $Y_p = 0.228 \pm 0.005$ (neglecting possible systematic errors). Adopting

a conservative upper bound $Y_p \lesssim 0.24$ in Eq. (4) gives then rise to the limit Eq. (2). Some quite recent observations of certain very low metallicity objects might however change the fit somewhat¹², and there are also some theoretical issues like the nucleon mass corrections on the reaction rates¹³, which also alter the computed value of Y_p slightly. It is nevertheless clear that in any case primordial nucleosynthesis precludes the appearance of extra degrees of freedom at the level of $\delta N_\nu \ll 1$.

The fit Eq. (4) can in fact be used to impose a simultaneous bound on the neutrino number density and the number of extra degrees of freedom. Let us denote the relative ν_e -abundance by n_{ν_e} and $\delta n_{\nu_e} \equiv n_{\nu_e} - 1$. Then, if g is the number of (fermionic) degrees of freedom additional to the Standard Model, one can show that nucleosynthesis requires that¹⁴

$$g - 4.6\delta n_{\nu_e} \leq \delta N_\nu , \quad (5)$$

with δN_ν deduced from observations as in Eq. (2) or Eq. (4). Electron neutrino density may change either due to oscillations or heavy particle decays, which take place after the decoupling.

3. Nucleosynthesis constraints on Dirac neutrinos

The production rate for the 'wrong helicity' neutrino ν_+ , and hence their mass, must be small enough so that it decouples already before the QCD phase transition which occurs at temperatures somewhere between 100 and 400 MeV. In that case they will not participate in the entropy transfer from quark-gluon plasma to particles in equilibrium, and consequently their number and energy densities will be diluted below levels that are acceptable for primordial nucleosynthesis. Assuming decoupling just above the QCD phase transition, at nucleosynthesis there would then remain a right-handed neutrino energy density which would be equivalent to about 0.1 neutrino species.

The 'wrong helicity' neutrino production rate was first estimated by Fuller and Malaney¹⁵, who argued that a Dirac neutrino with a lifetime exceeding the nucleosynthesis time scale ($t \sim 1$ s) should have a mass less than about 300 keV. A more detailed study of the scattering processes involved has recently been carried out in ref. 16.

The starting point is that before the QCD phase transition but below, say, $T \simeq 0.5$ GeV, the fermions present in the Universe at significant number densities were the leptons and u, d, s and c quarks. There are altogether 47 separate $2 \rightarrow 2$ reactions with (i) no 'wrong-helicity' neutrinos in the initial state and (ii) with at least one ν_+^μ or ν_+^τ in the final state, which all need to be taken into consideration. The constraint (i) is imposed on because each wrong helicity neutrino in the initial (final) state introduces an additional small factor $m_\nu^2/|\mathbf{p}|^2$ ($m_\nu^2/|\mathbf{p}'|^2$) to the cross section. Here $|\mathbf{p}|$ and $|\mathbf{p}'|$ are the absolute values of the centre-of-mass momenta of the incoming and outgoing particles, respectively. Hence processes with more than one 'wrong-helicity' neutrino can be ignored as compared to processes with only one 'wrong-helicity' neutrino. In addition, there are also quark and lepton decays which can produce ν_+ 's.

The thermally averaged scattering rate reads¹⁶

$$\Gamma_+^{sc} = \frac{1}{n_{\nu_+}^{eq}(T)} \sum_{(12 \rightarrow 34)} \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} f(E_1/T) f(E_2/T) \sigma_+^{(12 \rightarrow 34)} j(p_1, p_2), \quad (6)$$

where $n_{\nu_+}^{eq}$ is the equilibrium number density of ν_+ 's, $f(E_i/T)$ are the Fermi-Dirac distributions of the incoming particles, and $j(p_1, p_2) = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} / E_1 E_2$ is a flux-related factor. In Eq. (6) we have neglected the final state Pauli blocking, which is an about 10% effect¹⁶. Thermally averaged decay rate is simply given by

$$\Gamma_+^d = \frac{1}{n_{\nu_+}^{eq}(T)} \sum_{(1 \rightarrow 234)} \Gamma_+^{(1 \rightarrow 234)} \int \frac{d^3 p_1}{(2\pi)^3} f(E_1/T) \frac{m_1}{E_1}, \quad (7)$$

where the factor m_1/E_1 arises from the Lorentz boost of the decay rate.

Computing all the relevant processes in this approximation, and requiring that $\Gamma_+^{sc} + \Gamma_+^d < H$ at $T = T_{\text{QCD}}$, one finds¹⁷ for $T_{\text{QCD}} \simeq (100)200$ MeV that the mass limits are $m_{\nu_\tau} \lesssim (1180)740$ keV and $m_{\nu_\mu} \lesssim (720)480$ keV.

A similar line of argument can be used to set a limit on the magnetic moment of a light neutrino. The e^+e^- annihilation cross section for right-handed neutrinos in photon-mediated scattering has been estimated to be¹⁸

$$\sigma \simeq \frac{\mu_\nu \alpha^2 \pi}{6m_e^2} \left(\frac{1 - 4m_e^2/s}{1 - 4m_\nu^2/s} \right)^{1/2} \left(1 + 8 \frac{m_\nu^2}{s} \right) \left(1 + 2 \frac{m_e^2}{s} \right). \quad (8)$$

Demanding decoupling of the magnetic moment induced interactions prior to QCD phase transition yields the limit¹⁹

$$\mu_\nu \lesssim 5.2 \times 10^{-11} \mu_B \left(\frac{200 \text{ MeV}}{T_{\text{QCD}}} \right). \quad (9)$$

More stringent limits²⁰ will however be obtained from red giants and SN1987A. One may also put a nucleosynthesis limit on the neutrino charge radius²¹: $\langle r^2 \rangle \lesssim 10^{-32} \text{ cm}^2$.

Nucleosynthesis limits on Dirac neutrino masses, as a function of QCD phase transition temperature. The allowed region is below the curves. (From ref. 16).

The reasoning described above applies to neutrinos with a mass less than 1 MeV. A heavy (tau) neutrino with a mass in the MeV region would have a more pronounced effect on nucleosynthesis than a light neutrino, because during the synthesis of the light elements the energy density of the 'right-helicity' states of a heavy neutrino would be comparable to or higher than that of a massless neutrino. This is because at that epoch these states have already decoupled ($T_{dec}^{\nu-} \sim \text{few MeV}$). This has been shown²² to lead to an excluded region $0.5 \text{ MeV} \lesssim m_{\nu_\tau} \lesssim 30 \text{ MeV}$ for the tau neutrino mass, provided $\tau_{\nu_\tau} \gtrsim 10^3 \text{ s}$. (If $1 \text{ s} \lesssim \tau_{\nu_\tau} \lesssim 10^3 \text{ s}$, the upper bound is somewhat weakened.) This limit, with minor modifications, applies also to Majorana neutrinos.

If ν_τ has a very large magnetic moment, it will be kept in equilibrium by photon-mediated annihilations which help to decrease the ν_τ number density. In this way ν_τ could actually escape the nucleosynthesis constraint above. Following the relevant Boltzmann equations one finds¹⁸ in the mass range 5 – 35 MeV the bound $\mu_{\nu_\tau} \lesssim 10^{-8}$, with a slight mass-dependence. The bound is essentially determined by the upper limit on 4He .

4. Sterile neutrinos

If neutrinos have a non-zero mass, they might also mix with each other exactly like the quarks. In the early universe the mixing between flavour states is not expected to affect nucleosynthesis²³, because the number densities of different flavour states are (to a high accuracy) equal due to thermal equilibrium. If the flavour states however mix with a sterile neutrino ν_s , then many interesting effects arise. The initial number density of a sterile species may be assumed to be diluted by e.g. QCD phase transition, but oscillations may help to fill up the density back to its thermal level. Moreover, oscillations may deplete the ν_e population after the decoupling at $T \simeq 2.3$ MeV but above the n/p freeze-out at $T \simeq 0.7$ MeV. Both effects affect primordial nucleosynthesis, as is evident from Eq. (5).

The oscillation pattern between sterile and active neutrinos is in the heat bath of the early universe modified by the forward scattering of the active species. The average effective energy for ν_e at 1 MeV $\lesssim T \ll 100$ MeV is given by²⁴

$$V_e = \sqrt{2}G_F n_\gamma (L - AT^2/m_W^2), \quad (10)$$

where $A \simeq 55$ and L contains terms proportional to lepton and baryon asymmetries; if they are initially small enough, they will be dynamically driven to zero²⁵. Oscillation will not be effective if non-forward collisions destroy the coherence, which means that oscillations can start only at temperatures close to decoupling.

The oscillating system is conveniently described by a 2×2 one-body density matrix $\rho_\nu = \frac{1}{2}P_0(I + \mathbf{P} \cdot \sigma)$, and the equations of motion are²⁶

$$\begin{aligned}\frac{d\mathbf{P}}{dt} &= \mathbf{V} \times \mathbf{P} + (1 - P_z)\frac{d\ln P_0}{dt}\hat{\mathbf{z}} - (D + \frac{d\ln P_0}{dt})(P_x\hat{\mathbf{x}} + P_y\hat{\mathbf{y}}), \\ \frac{dn_{\nu_\alpha}}{dt} &= F_0[C_\alpha(1 - n_{\nu_\alpha}^2) - (n_{\nu_\beta}^2 - n_{\nu_\alpha}^2) - (n_{\nu_\gamma}^2 - n_{\nu_\alpha}^2)], \\ \mathbf{V} &= V_0(\sin 2\theta\hat{\mathbf{x}} - \cos 2\theta\hat{\mathbf{z}})\frac{\delta m^2}{eV^2}\frac{\text{MeV}}{T} - V_{\nu_\alpha}\hat{\mathbf{z}}\left(\frac{T}{\text{MeV}}\right)^5.\end{aligned}\tag{11}$$

Here $\alpha \neq \beta \neq \gamma \subset \{e, \mu, \tau\}$, $C_e = 2.31$, $C_\mu = C_\tau = 0.51$ and $F_0 = 2.65 \times 10^{-2}(T/\text{MeV})^5\text{s}^{-1}$. V_0 and V_{ν_α} are the effective energies, generalized from Eq. (10) to include also changes in n_{ν_e} , D the damping rate and the equation of motion for P_0 is simply $dP_0/dt = dn_{\nu_\alpha}/dt$. The number densities are normalized such that $n_{\nu_\alpha} = 1$ corresponds to a single neutrino degree of freedom in chemical equilibrium. A complete description of the evolution of the oscillating system Eq. (11) requires numerical study which reveals²⁶ that the large-angle MSW solution for the solar neutrino problem²⁷ is, in the case of $\nu_e - \nu_s$ mixing, ruled out by nucleosynthesis. Similarly, the nucleosynthesis constraint rules out²⁸ $\nu_\mu - \nu_s$ oscillations as an explanation for the atmospheric neutrino puzzle²⁹. (For a recent investigation of sterile-active oscillation in the early universe, including a numerical nucleosynthesis code, see ref. 30.)

Sterile neutrinos has also been proposed as a candidate for the cold component in a cold and hot mixture of dark matter, which has become popular in the view of the COBE detection³¹ of the anisotropy in the microwave background. It has been argued³² that sterile neutrino with a Majorana mass in the range 0.1-1.0 keV, slightly mixed with an ordinary neutrino, would provide warm dark matter and structure formation with more power on small scales than in hot dark matter scenarios.

Another interesting possibility, first pointed out by Madsen³³, is to have a heavy fermion ν_H which decays into a light sterile fermion and a light boson. These may then be assumed to have decoupled prior to QCD phase transition. If ν_H decay while they are still relativistic, solving for the relevant Boltzmann equations one finds that³⁴ equilibration in the final state will be preceded by decays into very cold ($p \ll T$) bosons. The bose condensation is effective provided the decay rate is fast enough, or equivalently, if the decay temperature is large enough: $T_d > \sqrt{m_H m_B}$ where m_B is the boson mass. This would then

be a natural way to generate a mixture of hot and cold dark matter. Numerical studies show³⁴ that one obtains about 40% cold bosons, but this could still not be enough as the favoured cold/hot ratio appears to be³⁵ 70/30.

5. Spin rotation in magnetic fields

Right handed neutrinos may also be produced by scattering of ν_L off a primordial magnetic field, if such exists. This provides an interesting connection with neutrino properties and primordial nucleosynthesis on one hand, and with the observed galactic magnetic fields on the other hand. The galactic magnetic fields, which are of the order of few μG , are believed to have arisen from a very weak seed field through the so-called galactic dynamo mechanism³⁶. The differential rotation and turbulent motion inside a galaxy amplifies the seed field exponentially until a saturation point is reached. The field is observed indirectly by measuring the synchrotron radiation of the electrons which traverse the galactic field, assuming equipartition of magnetic and particle energies (this assumption has however recently been subject to some discussion³⁷). Observationally not much is known about the seed field. Some weak bounds on it may be obtained by requiring that the growth time must be longer than galactic rotation period. The observation³⁸ of the magnetic field in a spiral galaxy with $z=0.395$ seems to indicate that the dynamo was saturated already some time ago, implying a relatively large seed field. Moreover, the magnetic field of the Milky Way changes its direction by about 180° between the Sagittarius and Orion arms, which has been argued³⁹ to be an indication for a large seed field of the order of about 10^{-11} G. In computer simulations⁴⁰ seed fields of the order of 10^{-18} seem to be able to produce the observed field strength (but not the reversals, which however has been observed only in the Milky Way).

It has been argued⁴¹ that a random seed field of the correct size will indeed be produced by fluctuations in the Higgs field at the electroweak phase transition. The point is that at the electroweak phase transition all the physical quantities should be uncorrelated over distances greater than the horizon distance. This means in particular that the Higgs field cannot be gauge rotated to point into same direction in group space in every horizon volume⁴². Therefore there will appear physical Higgs field gradients from which one may construct an electromagnetic field $F_{ij} = -i(V_i^\dagger V_j - V_i V_j^\dagger)$ with $V_i = 2\sqrt{\sin\theta_W/g}\partial_i\phi/|\phi|$. This

is a random field, frozen to the charges in the primeval plasma, and for any Gaussian distribution one finds⁴¹

$$B_{rms} \equiv \sqrt{\langle B^2 \rangle} = B_0 \left(\frac{R_0}{R(t)} \right)^2 \frac{1}{\sqrt{N}}, \quad (12)$$

where N is the number of correlation lengths and $B_0 \simeq m_W^2 \simeq 10^{24}$ G. This yields today and at the intergalactic distances of 100 kpc a root mean square field of 4×10^{-19} G with $\langle B \rangle = 0$.

The direct cosmological consequences of the random field Eq. (12) are expected to be minor⁴¹, except possibly for Dirac neutrino spin flip and hence nucleosynthesis. For instance, the magnetic energy density at the horizon scale is much smaller than radiation energy density. At the onset of nucleosynthesis, and at the horizon scale, the magnitude of the field is about $B_{rms} \simeq 1500$ G. This is well below the nucleosynthesis limit⁴³ $B \lesssim 10^{11}$ G on primordial magnetic field, which is based both on the effects on the expansion rate and on nuclear reactions rates.

To tackle neutrino helicity change in an external slowly varying random magnetic field it is best to use a kinetic equation⁴⁴ for the Wigner neutrino spin distribution function $\mathbf{S}(\mathbf{p}, \mathbf{x}, t)$. It is also useful to consider the combination

$$\begin{aligned} \tilde{H}_\perp(t) e^{\pm i\alpha(t)} &= \mu_\nu (B_x(t) + E_y(t) \pm i(B_y(t) - E_x(t))) \\ &= \frac{\mu_\nu}{\sqrt{2}} \int \frac{d^3k}{(2\pi)^3} [\hat{B}_x(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{B}_x^\dagger(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} \pm i(\hat{B}_y(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{B}_y^\dagger(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}})], \\ \alpha(t) &= \arctan\left(\frac{B_y(t) - E_x(t)}{B_x(t) + E_y(t)}\right), \end{aligned} \quad (13)$$

corresponding to neutrino propagation along z -axis so that

$$\tilde{H}_\perp(t) = \mu_\nu \sqrt{B_x^2(t) + B_y^2(t) + E_x^2(t) + E_y^2(t) + 2[E_y(t)B_x(t) - B_y(t)E_x(t)]}.$$

The basic starting point for studying neutrino spin flip in random magnetic field is then the neutrino spin kinetic equation for the z -component of the neutrino spin $\mathbf{S}(t) = \int d^3p \mathbf{S}(\mathbf{p}, t)$, which takes the form⁴⁴

$$\begin{aligned} \frac{dS_z(t)}{dt} &+ 2\tilde{H}_\perp(t) e^{i(\alpha-Vt)} \int \tilde{H}_\perp(t) e^{-i(\alpha-Vt)} S_z(t) dt \\ &+ 2\tilde{H}_\perp(t) e^{-i(\alpha-Vt)} \int \tilde{H}_\perp(t) e^{i(\alpha-Vt)} S_z(t) dt \\ &= i \left(C_{-1} \tilde{H}_\perp(t) e^{-i(\alpha-Vt)} - C_{+1} \tilde{H}_\perp(t) e^{i(\alpha-Vt)} \right), \end{aligned} \quad (14)$$

and which depends on the fluctuating magnetic field squared. The constants $C_{\pm 1}$ are determined from boundary conditions, and \tilde{H}_\perp , $\alpha(t)$ are given by Eq. (13). $V = 3.45 \times 10^{-20} (T/\text{MeV})^5 \text{ MeV}$ is the effective neutrino energy as obtained from Eq. (10) with $L = 0$.

To make use of the complicated kinetic equation (14) one has to average over the fluctuations. This can be achieved by assuming isotropy and using Wick rules for the various averages⁴⁵:

$$\begin{aligned}\langle \hat{B}_i^\dagger(\mathbf{k}) \hat{B}_j(\mathbf{k}') \rangle &= (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \langle B_i^\dagger B_j \rangle, \\ \langle B_i^\dagger B_j \rangle_{\mathbf{k}} &= (\delta_{ij} - k_i k_j / k^2) \langle \mathbf{B}^2 \rangle_{\mathbf{k}}.\end{aligned}\tag{15}$$

Many of the terms in Eq. (14) actually vanish by virtue of the averaging procedure. After some manipulations, one finally finds that the kinetic equation reduces to the simple expression⁴⁵

$$\frac{d^3 S_z(t)}{dt^3} + \omega_0^2 \frac{dS_z(t)}{dt} = 0,\tag{16}$$

where the neutrino spin rotation frequency ω_0 is given by

$$\omega_0 = \sqrt{(\dot{\alpha} - V)^2 + 4\tilde{H}_\perp^2}.\tag{17}$$

Solving Eq. (16) with the appropriate boundary conditions one immediately obtains

$$P_{\nu_L \leftrightarrow \nu_R} = \frac{1 + S_z(t)}{2} = \frac{4\tilde{H}_\perp^2}{\omega_0^2} \sin^2(\omega_0 t/2),\tag{18}$$

where the frequency now reads

$$\omega_0 = \sqrt{V^2 + 8\langle \tilde{H}_\perp^2 \rangle + \frac{6}{5}L^{-2}}.\tag{19}$$

Here L is a measure of the magnetic field energy density inhomogeneity, and it is determined by the ratio

$$L^{-2} = \frac{\int k^2 \langle \mathbf{B}^2 \rangle_k d^3k / (2\pi)^3}{\langle \mathbf{B}^2 \rangle_{\mathbf{x}=0}}.\tag{20}$$

A small fluctuation length of the magnetic field thus effectively damps the neutrino to spin flip probability. Unfortunately, it is not clear what a realistic

coherence length of the primordial field should be. Let us however assume that above the QCD phase transition the field is coherent over length scales of the order of the (electron) neutrino scattering length $L_W = (4.0G_F^2 T^5)^{-1} \simeq 3 \times 10^{-7} l_H$ at $T = 200$ MeV (l_H is the horizon length). We may then average $P_{L \rightarrow R}$ over L_W to obtain the production rate. One finds that right-handed neutrinos would be in equilibrium at nucleosynthesis unless

$$\Gamma_{\nu_L \rightarrow \nu_R} = \langle P_{L \rightarrow R} \rangle L_W^{-1} \lesssim H(T = T_{\text{QCD}}). \quad (21)$$

In essence, the spin content of the thermal neutrino bath is thus tested by ordinary weak collisions. From Eq. (18) one obtains the bound

$$\mu_\nu B(T = 200 \text{ MeV}, L_W) \lesssim 7 \times 10^2 \mu_B G. \quad (22)$$

Assuming the scale dependence of the coherent field is known, we can find the magnitude of the galactic seed field at T_{QCD} . With the scaling Eq. (12) one would then obtain the limit $\mu_\nu \lesssim 2 \times 10^{-10} \mu_B$; with $B \sim 1/N$ the limit would be⁴⁶ $\mu_\nu \lesssim 6.5 \times 10^{-34} \mu_B G / B_{\text{seed}}(T_{\text{now}}) \simeq 2 \times 10^{-16} \mu_B$. (A similar constraint applies also for transition magnetic moments). In the Standard Model this argument can also be turned the other way round to yield a limit on the magnetic field strength at QCD phase transition, provided galactic dynamo is the explanation for the observed galactic magnetic fields:

$$B(T = 200 \text{ MeV}) \lesssim \frac{10^{21} \text{ G}}{\sum (m_{\nu_i} / \text{eV})}. \quad (23)$$

Again depending on the scaling of the magnetic field to intergalactic distances, Eq. (23) provides a limit on the sum of all neutrino masses⁴⁷.

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